It follows that the coefficient of the second leading term gives the sum of the roots times a_3, and that the constant term gives the product of the roots times a_3. Dividing both expressions by a_3 yields exactly what our formulas above say.

Example 2.5. Assume we know that one of the roots of the polynomial $f(x)=x^3-11x+c$ is $2+i$. We want to find the value of c and the other roots of $f(x)$.

This problem is similar to worked out problem 2 in section 2.2, and thus it could be solved in the same way. However, we want to solve this problem using the results obtained in this section.

Since $2+i$ is one of the roots, the complex conjugate root theorem forces $2-i$ to be a root of $f(x)$ as well. Let γ be the third root of $f(x)$. Since the polynomial has degree 3, these are the only roots. Now we use what we have learned in this section to get

$$(2-i)+(2+i)+\gamma=\frac{-0}{1}=0 \qquad (2-i)(2+i)\gamma=(-1)^3\frac{c}{1}=-c$$

The first equation can be re-written as $4+\gamma=0$, and so $\gamma=-4$. Plugging this into the second equation, we get $(2-i)(2+i)(-4)=-c$. Multiplying all that out, we get $c=20$.

Exercises

2.1. Factor the polynomial $f(x)=x^3-2x^2-5x+6$ and plot the graph.

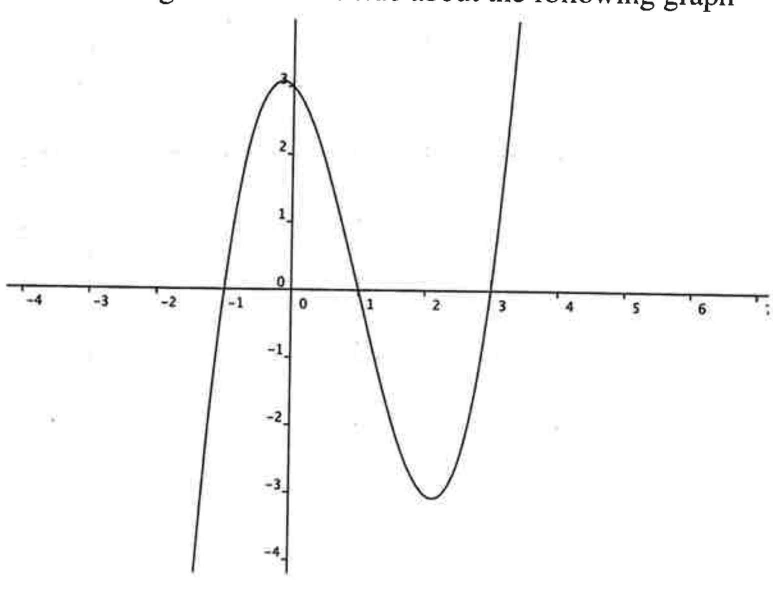
2.2. Factor the polynomial $f(x)=x^3-3x^2-x+3$ and plot the graph.

2.3. Does the polynomial $f(x)=x^9+1$ have any rational roots? Justify your answer.

2.4. Does the polynomial $f(x)=x^9+5x^4+2$ have any rational roots? Justify your answer.

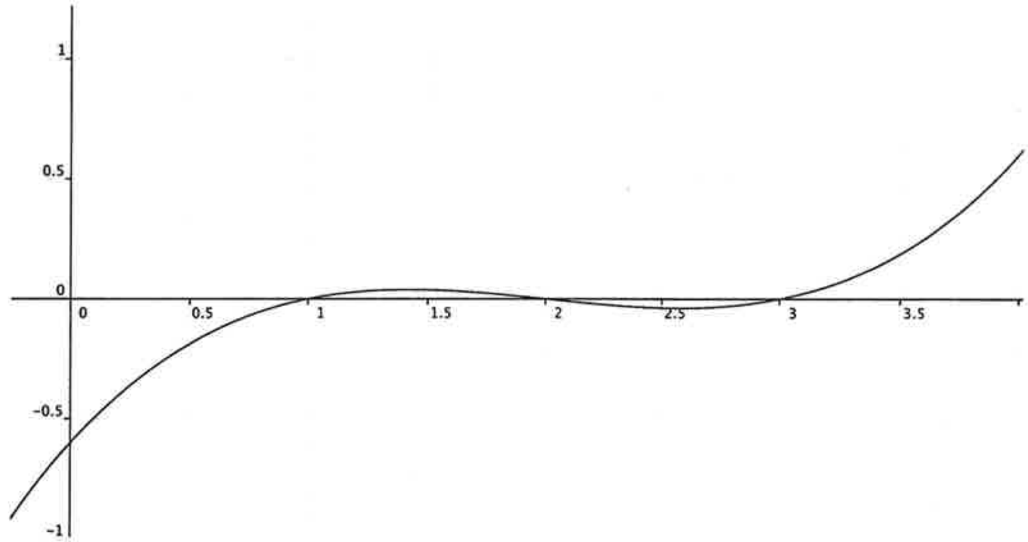
2.5. Find a rational root of $f(x)=3x^3-2x^2+3x-2$, then factor it completely and draw its graph.

2.6. Which of the following statements is true about the following graph



a. The polynomial has at least one complex root.
b. The polynomial has $(x-3)$ as a factor.
c. The polynomial is divisible by x^2+4x+3.

2.7. Which of the following statements is true about the following graph



a. The polynomial has at least one complex root.
b. The polynomial has $(x+1)$ as a factor.
c. The polynomial is divisible by x^2-4x+3.

2.8. Completely factor and draw the graph of $f(x)=2x^3-3x^2-9x+10$.

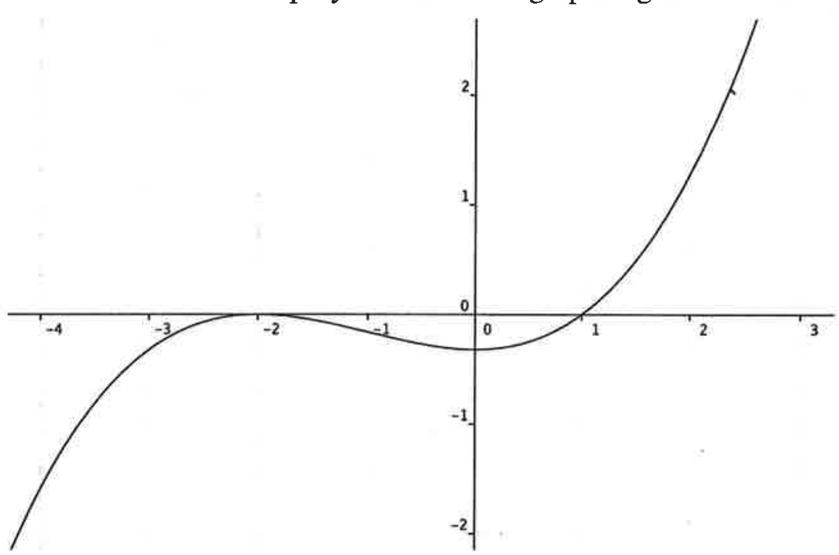
2.9. Completely factor and draw the graph of $f(x)=4x^3+6x^2+2x+3$

2.10. Completely factor and draw the graph of $f(x)=-x^3+5x^2-5x+1$

2.11. Completely factor and draw the graph of $f(x)=x^4+3x^3-11x^2-3x+10$

2.12. Completely factor and draw the graph of $f(x)=x^3-3x^2+4$

2.13. What must be true about the polynomial whose graph is given below?

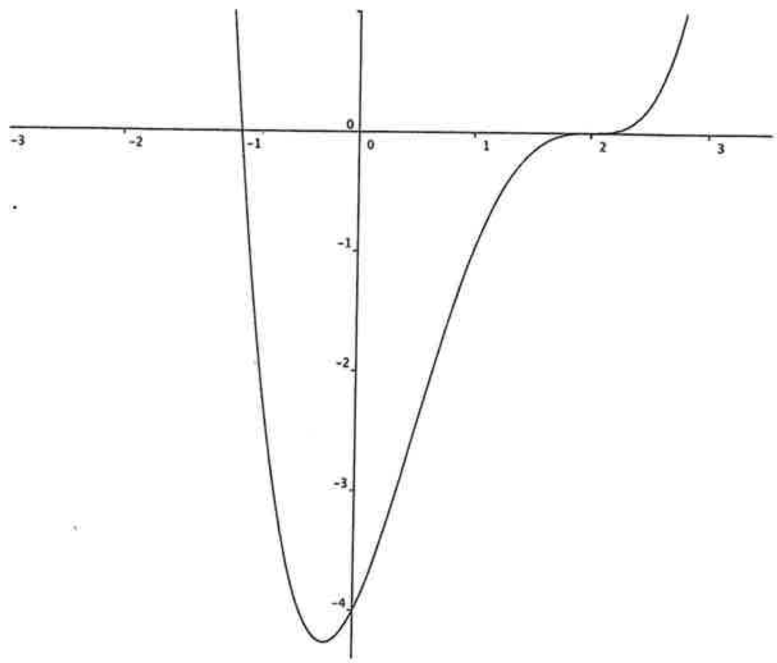


a. The polynomial has no complex roots.
b. The polynomial has a multiple root at $x = -2$.
c. The polynomial must be cubic.

2.14. Completely factor and plot the polynomial $f(x) = x^4 - 5x^3 + 6x^2 + 4x - 8$.

2.15. Completely factor and plot the polynomial $f(x) = x^4 - 2x^3 + 2x - 1$.

2.16. What must be true about the polynomial whose graph is given below?



a. The polynomial must be of degree 2.
b. The polynomial must have non-real roots.
c. The polynomial has no repeated root.
d. None of the above.

2.17. You are given that one of the roots of the polynomial $f(x) = x^3 - x + c$ is $1 + i\sqrt{2}$. Find c, the other roots of $f(x)$, and plot the graph of $f(x)$.

2.18. You are given that one of the roots of the polynomial $f(x) = x^3 - 4x^2 + 8x + c$ is $1 + i\sqrt{3}$. Find c and the other roots of $f(x)$.

2.19. Find the largest value of b such that all the roots of $f(x) = x^3 + 3x^2 + 5x + 3$ are of the form $-1 + ib$.

2.20. A polynomial $f(x)$ has roots -3, $\sqrt{2}$, and $5i$. What is the smallest degree it can have?

2.21. Is it possible for a polynomial of degree 5 to have 3 imaginary roots and two real roots?

2.22. You are given that two of the roots of $f(x) = x^5 + x^4 + x^3 + x^2 - 12x - 12$ are $\sqrt{3}$ and $-2i$

a. Name other irrational and imaginary rots that $f(x)$ can have.
b. What are the possible integer roots of the equation?
c. Out of these possible roots, how many could $f(x)$ actually have?